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Effect of High Frequency Oscillation on the Photo-electric Current

by Tatsumi Terada

(Received 19 February 1938)

Abstract

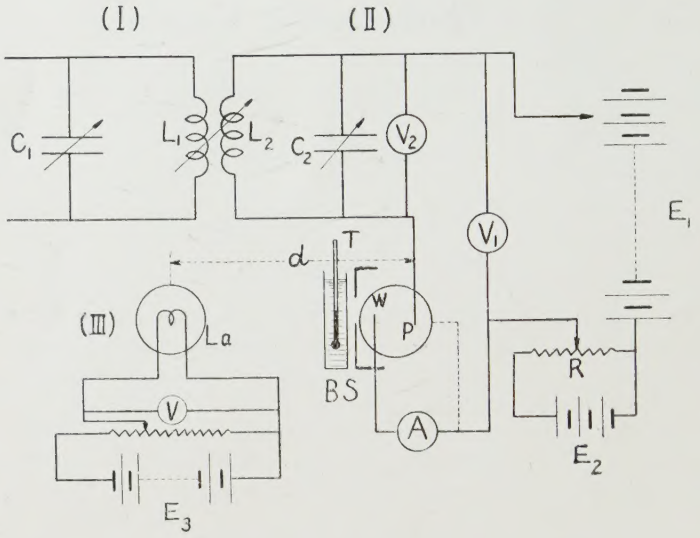
The electric current from a photo-cell increases when a very high frequency oscillation of a few volts is superposed upon a continuous voltage. This peculiar effect depends upon the intensity of illumination and the frequency of the oscillatory voltage, but is nearly independent of the temperature. The effect may be explained at least partly by the disturbances of the negative charge accumulated in front of the cathode of the cell. From this point of view, it is suggested that a critical frequency of the electrons in the cell is about 2140 kilocycles per sec.

It was found by Shenstone[1] that the photo-electric current from a plate of bismuth or of other metals was temporarily increased by the passage of an electric current, and it was attributed by him to an orientation of the elementary crystals of the metals. Horton[2] ascribed the effect to the emission of gases from those portions of the apparatus which were warmed by the current through the metals. Recently, it was concluded by Weber[3] that the "Shenstone effect" must be attributed to the removal of occluded gases from the metals.

It was found by the writer that the current from a photo-cell also increased when a continuous voltage was superposed by high frequency oscillation of a few volts. To investigate this peculiar effect, an apparatus shown in Fig. 1 was employed. (*I*) is a high frequency oscillatory circuit and (*II*) a photo-electric circuit. They are coupled by the coils L_1 and L_2. The oscillatory voltage induced in the circuit (*II*) is altered by changing the coupling and it is measured by means of Moullin's thermionic voltmeter V_2 whose scale is calibrated for every frequency before the measurements are made. A is a microammeter, V_1 a direct current voltmeter, and R is a stretched wire potentiometer of 30 ohms to be used for the fine adjustment of the continuous voltage

1. A. G. Shenstone: Phil. Mag. **41**, 916 (1921); **42**, 596 (1921), **45**, 918 (1923).
2. F. Horton: Phil. Mag. **42**, 279 (1921).
3. A. H. Weber: J. Frank. Inst. **223**, 215 (1937).

Fig. 1.



V_1. (*III*) is an electric circuit for light source, containing a gas-filled tungsten lamp L_a and a voltmeter V. The light passes through a water-vessel B having the size of 0.9 cm. in width and 12 cms. in length, and a circular slit S 4 cms. in diameter opening at a wall of a soft iron box and then it falls on the cathode P. Consequently, the heating of the cathode by the light is mostly eliminated by the water in the vessel B; and the stray magnetic field is partly screened by the soft iron box.

A voltage-current curve for a caesium vacuum photo-cell (Mazda PG-65-V) used in most of the experiments is shown in Fig. 2. It was obtained by varying the continuous voltage V_1 without applying any oscillatory voltage, when the cathode P was illuminated by the light of constant intensity emitted from a 60 watt-lamp placed at a distance of 30 cms. from the cell. In the figure, the abscissa gives the continuous voltage V_1 in volts, and the ordinate the current i_0 in microamperes.

However, when a very high frequency oscillatory electromotive force V_2 was superposed upon the continuous voltage V_1, the photo-electric current generally differed from the value i_0 corresponding to the case employing only a continuous voltage V_1 in Fig. 2. Some observations were made with several constant continuous voltages V_1, by varying the oscillatory voltages V_2 having the constant wave-length 140 ms. In Fig. 3, values obtained of the photo-electric current i are

Fig. 2.

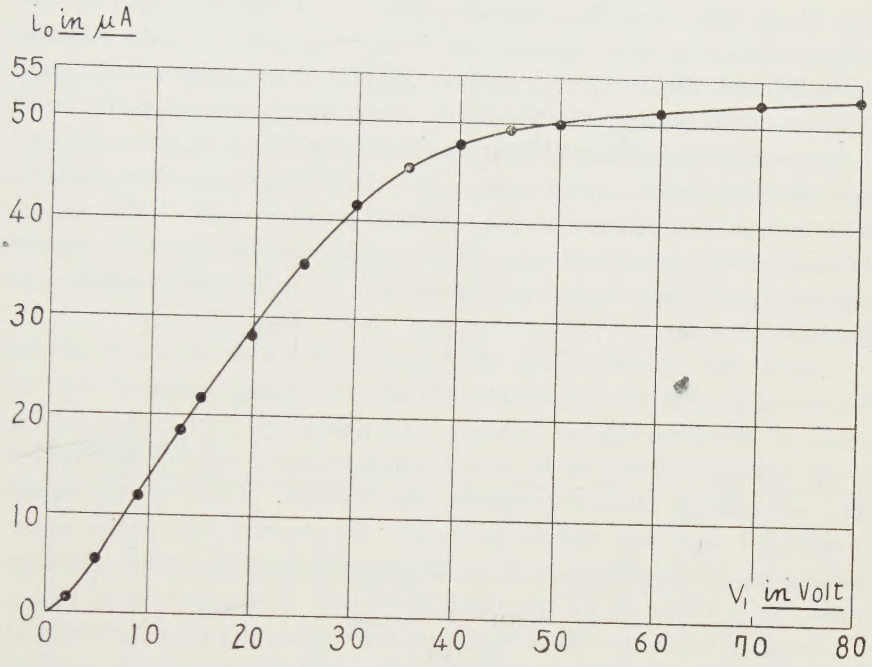
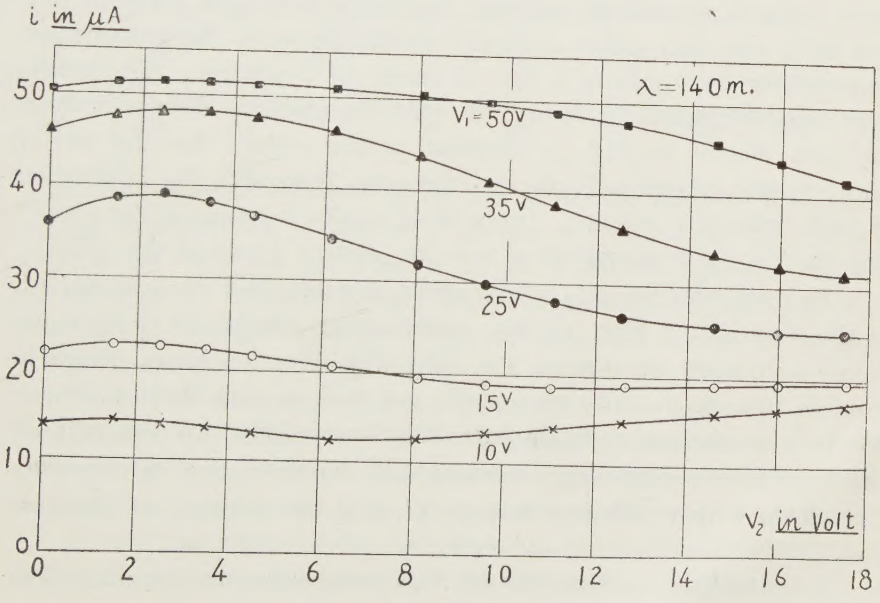

i_0 in μA
55
50
40
30
20
10
0
0
10
20
30
40
50
60
70
80
V_1 in Volt

Fig. 3.


i in μA
λ=140m.
V_1=50V
35V
25V
15V
10V
50
40
30
20
10
0
0
2
4
6
8
10
12
14
16
18
V_2 in Volt

plotted as ordinate against the corresponding values of oscillatory voltage V_2 as abscissa. From these curves, it will be seen that at first the current i increases up to a maximum and then begins to decrease gradually with the oscillatory voltage greater than about 3 volts, and also that the increase of the current is much more marked in the case of the continuous voltage $V_1 = 25$ volts, though as will be mentioned later on, it depends upon the intensity of illumination. The peculiar increase was not limited to the vacuum caesium cell. The results obtained with a gas-filled caesium cell and a gas-filled potassium cell were found to give similar curves, though the effect differed more or less in magnitude.

As to the causes for the effect described above, it seems almost inconceivable that the rectification of the cell is the cause of the increase of the photo-electric current. As shown in Fig. 2, the portion of the voltage-current curve which corresponds to the voltages higher than 25 volts is concave towards the V_1-axis. Thus, if the photo-electric cell used acts simply to rectify, the current due to the superposition of the oscillatory voltage V_2 should decrease rather than increase, as is revealed by the right hand part of the curves in Fig. 3.

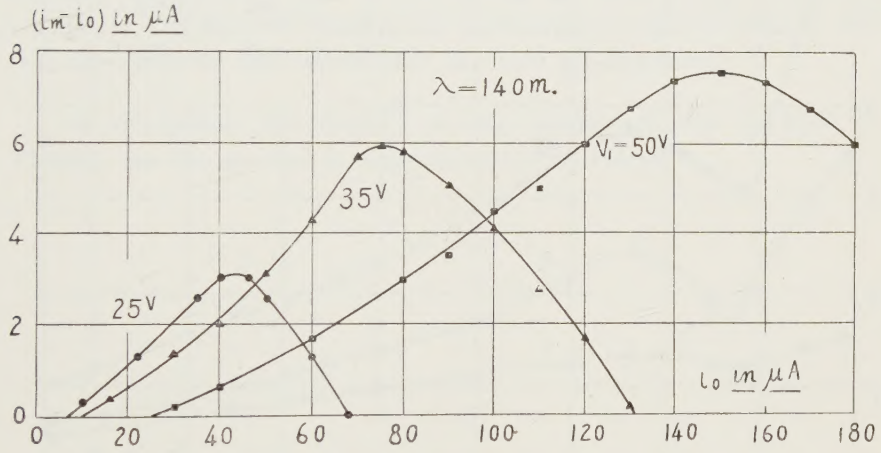
Next, to know the effect of temperature upon the peculiar increase, another experiment was carried out. The cell was placed in a closed vessel with an automatic regulating heater; and similar observations were repeated at various constant temperatures ranging between 10°C and 60°C, and also under controlled conditions while the temperature increased or decreased at a rate of about 0.5°C per min. The results thus obtained indicated that there was no essential difference from the case shown in Fig. 3. Further, it was noticed that the rise of photo-electric current took place at the same time with the application of high frequency voltage. From these results, it seems almost certain that the rise of temperature is not the primary cause of the effect.

To learn whether the peculiar effect depends upon the wave-length of the illuminating light or not, experiments similar to those made before were made by filtering the light with Wratten filters placed in front of the slit S. The results did not indicate any distinct connection between the wave-length of the illuminating light and the peculiar effect under consideration. It was found, however, that the effect depended upon the continuous voltage V_1 and the intensity of illumination.

To investigate qualitatively the relation between the peculiar effect

and the intensity of illumination, the following measurements were made. The photo-electric current i_0 obtained with a constant continuous voltage V_1 increases with the intensity of illumination. Therefore, by keeping the continuous voltage V_1 constant the photo-electric current i_0 due to V_1 was made to vary by changing the distance between the lamp and the photo-cell; and for different values of i_0 the maximum currents i_m obtained by superposing various oscillatory voltages V_2 of constant frequency to V_1 were measured. Some of the results obtained with oscillatory voltages having the wave-length 140 ms. are plotted in the curves of Fig. 4, and it is to be noticed that there is a crest on each curve. Take the curve of $V_1=25$ V as an example.

Fig. 4.



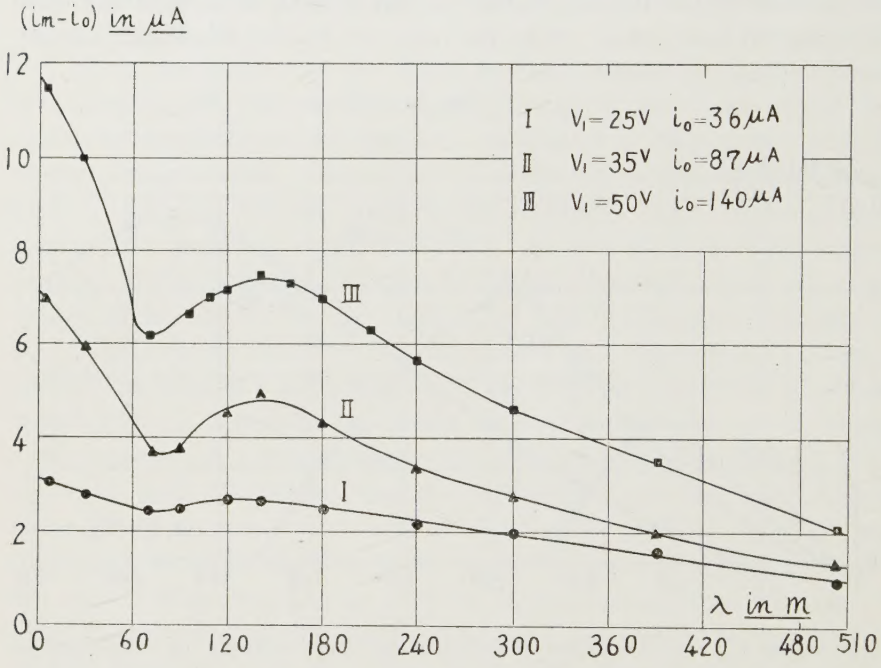
The value (i_m-i_0) becomes maximum at the intensity of illumination $i_0=42$ microampers ($d=29$ cms. for the 60 watt-lamp), the increase-ratio at the crest $(i_m-i_0)/i_0$ being 7.5%. However, when the intensity of illumination becomes greater the increase-ratio reduces, and at an illumination stronger than $i_0=60$ microampers ($d=15$ cms. for the same lamp), it changes into negative.

Since the number of electrons emitted from the cathode is directly proportional to the intensity of the illuminating light, as was verified over a wide range, we can say that the effect under consideration has an important connection with the number of the emitted electrons.

Lastly, the effect of the frequency of the oscillating voltage was investigated. In such experiment the intensity of illumination i_0 and

the continuous voltage V_1 were kept constant; and the maximum currents i_m obtained by applying various oscillatory voltage V_2 having different frequencies were measured. The results obtained are shown in the curves of Fig. 5. It is to be noticed that the peculiar effect $(i_m - i_0)$ generally decreases with the increase of the wave-length; for example, it disappeared at $\lambda = 1000$ ms. for the curve *I*, and that a maximum appeared in each curve at the wave-length of about 140 ms. The

Fig. 5.



results obtained with the other photo-cell also were found to give similar curves.

From the facts stated above, it seems probable that the peculiar increase of the photo-current is at least partly due to the disturbances caused by the oscillatory voltages upon the negative charge accumulated in front of the cathode when the photo-electric current is smaller than that of saturation: the electrons arriving at the anode will become more in number when such accumulated negative charge is disturbed by the oscillatory voltage. If so, the peculiar effect should be more remarkable at the oscillatory voltage whose frequency is in resonance

with that of the electrons and the critical frequency of the electrons in the cell is found to be about 2140 kilocycles (wave-length 140 ms.) in the present case. About ten years ago, Nichols and Schelleng[1] published a paper as to the propagation of radio waves and concluded that the critical frequency of the electrons in the air was 1400 kilocycles (214 ms.). After that, Deutsch[2] observed that the electrons in an atmospheric pressure were in resonance with the wave-length of about 50 ms. The critical wave-length obtained by the writer is shorter than that of Nichols and Schelleng, but longer than that of Deutsch.

The fact that the peculiar effect is not marked when the initial photo-electric current i_0 comes near to the saturation current may be explained simply by the scanty accumulation of the negative charge in front of the cathode. The decrease of the value $(i_m - i_0)$ for the stronger illumination may be due to the fact that greater oscillatory voltage is necessary to disturb the accumulated negative charge, and that the effect of the rectification becomes predominant.

In conclusion the writer's sincere thanks are due to Prof. U. Yoshida for the interest he has taken in the research.

1. H. W. Nichols and J. G. Schelleng: Nature, **115**, 334 (1925).
2. W. Deutsch: Ann. d. Physik, **26**, 195 (1936).

On the Proof of Carleman's Extension Theorem of Liouville Theorem

by Tunezô Satô

(Received 12 April, 1938)

As an extension theorem of Liouville theorem, Torsten Carleman proved two theorems[1] as follows:

I. *Let* $\omega(\varphi)\geqq 0$, *and*

$$\int_0^{2\pi}\omega(\varphi)d\varphi<K \qquad (\textit{finite}),$$

then an integral function $f(z)$ *which satisfies the inequality*

$$|f(z)|<e^{e^{\omega(\varphi)}} \qquad (z=re^{i\varphi})$$

should be constant;

II. *Let* $\omega(\varphi)\geqq 0$ *and* $\omega(\alpha)$, $\omega(-\alpha)$ *be finite. Let*

$$\int_{-\alpha}^{\alpha}\omega(\varphi)d\varphi<K.$$

Then a function $f(z)$, *which is regular and analytic in the angular domain* $-\alpha<\varphi<\alpha$ *and satisfies the inequality*

$$|f(z)|<e^{e^{\omega(\varphi)}},$$

should be bounded throughout the angular region $-\alpha\leqq\varphi\leqq\alpha$.

In this paper we wish to prove these theorems most briefly by a different method from the one by which Carleman followed.

Let $f(z)$ be regular and analytic for $|z|<R$, then the integral of F and R. Nevanlinna

$$(1) \qquad M(r)=\frac{1}{2\pi}\int_0^{2\pi}\log^{+}|f(re^{i\theta})|\,d\theta \qquad (r<R)$$

is a function which is convex and non-decreasing with respect to $\log r$.[2] And besides if, for any two numbers r_1, r_2 such as $r_1<r_2<R$, $M(r_1)=M(r_2)$, the function $M(r)$ will be constant by virtue of its convexity. Now first let $M(r)=0$. Then for every $r(<R)$ we have

$$\frac{1}{2\pi}\int_0^{2\pi}\log^{+}|f(re^{i\theta})|\,d\theta=\frac{1}{2\pi}\int_{|f|\geqq 1}\log|f(re^{i\theta})|\,d\theta=0.$$

1. Acta Math., **48** (1926), pp. 363–366.
2. P. Montel; Jordan Jour., IX(1928), pp. 29–60.

Since the integrand is not negative, unless for all r the Lebesgue measure of the set $E(|f|\geqq 1)$ is zero,[1] almost all values of θ in $E(|f|\geqq 1)$

$$\log^{+}|f(re^{i\theta})|=0 \quad i.\ e.\ \ |f(re^{i\theta})|=1.$$

Hence from the regularity of $f(z)$, we have $f(z)=const.$ throughout $|z|<R$. Second, let $M(r)=c$ (>0). For the present case we take c_0 (>1) such as $e^c=c_0$. Then we have

$$\frac{1}{2\pi}\int_0^{2\pi}\log^{+}|f(re^{i\theta})|\,d\theta-\frac{1}{2\pi}\int_0^{2\pi}\log c_0\,d\theta\geqq\frac{1}{2\pi}\int_0^{2\pi}\log^{+}|c_0 f(re^{i\theta})|\,d\theta=0;$$

because, as $|c_0 f|\geqq|f|$, the set $E(|c_0 f|\geqq 1)$ may be contained in the set $E(|f|\geqq 1)$. Thus by the same argument as in the first case, it is very easy to show that $f(z)=const.$

Now from the above stated properties of the function $M(r)$ we shall prove Carleman's Theorem I.

By the formula of Schlömilch[2] we have

$$\int_0^{2\pi}e^{\omega(\varphi)}d\varphi\leqq\exp.\left[\int_0^{2\pi}\omega(\varphi)d\varphi\Big/\int_0^{2\pi}e^{\omega(\varphi)}d\varphi\right],$$

$$i.\ e.\qquad\left[\int_0^{2\pi}e^{\omega(\varphi)}d\varphi\right]^{\int_0^{2\pi}e^{\omega(\varphi)}d\varphi}\leqq\exp.\int_0^{2\pi}\omega(\varphi)d\varphi;$$

but by the hypothesis, $\int_0^{2\pi}e^{\omega(\varphi)}d\varphi<const.$ While $\log^{+}|f(z)|<e^{\omega(\varphi)}$; hence, by (1), where $R\to\infty$, the function $M(r)$ is bounded for all values of r. On the other hand from the convexity of $M(r)$, unless $M(r)=const.$, it should be that $M(r)\to\infty$ as $r\to\infty$. Therefore necessarily $M(r)=const.$ for every r. This completes Theorem I.

In the sequel, for the purpose of the proof of Theorem II we define $F(z)$ as follows:

$$F(z)=f(z)\ [-\alpha\leqq\arg z\leqq\alpha];\ =0\quad[\text{elsewhere}].$$

Then the function $\log^{+}|F(z)|$ is subharmonic. Hence from the theory of subharmonic functions by Montel,[3] an analogous integral with the one of Nevanlinna

$$(2)\qquad M_1(r)=\frac{1}{2\pi}\int_0^{2\pi}\log^{+}|F(re^{i\theta})d\theta=\frac{1}{2\pi}\int_{-\alpha}^{\alpha}\log^{+}|f(re^{i\theta})|\,d\theta$$

must hold the same properties as $M(r)$. Thus by the hypotheses of theorem II, we have $M_1(r)=const.$ for every r.

Now let $M_1(r)=0$. Then if for a certain value of r it occurs

1. For this case our theorem becomes trivial by the classical Liouville Theorem.
2. See, *e. g.*, J. L. W. V. Jensen; Acta Math., t. **30** (1906), p. 178.
3. *Loc. cit.*

that $|f|>1$, we shall have $f(z)=const.$ throughout the angular region $-\alpha\leqq\varphi\leqq\alpha$; but otherwise we must have $|f(z)|\leqq 1$ throughout $-\alpha\leqq\varphi\leqq\alpha$.

Now since we can similarily discuss also the case where $M_1(r)=c(>0)$, it follows that Theorem II is correct.

In conclusion the author wishes to express his hearty thanks to Prof. Toshizô Matsumoto for his kind remarks during the study.

On the Condition of the Set which might be a Derived Set

By **Tameharu Shirai**

(Received April 28, 1938)

"*Let E be a metric space and M an isolated set contained in E_β[1], then there exists a set A such that*

$$M_\alpha[2]=A_\beta.$$"

For any point $x\varepsilon M$, there are neighbourhoods of x which contain no points of M except x itself. We shall denote the upper bound of the radii of these by ρ_x (>0), then we have

$$U_x\left(\frac{\rho_x}{2}\right)U_{x'}\left(\frac{\rho_{x'}}{2}\right)=0 \qquad \text{for } x\neq x' \ (x\varepsilon M,\ x'\varepsilon M).$$

Because, if $U_x\left(\frac{\rho_x}{2}\right)$ and $U_{x'}\left(\frac{\rho_{x'}}{2}\right)$ have a common point y, it follows that $yx<\frac{\rho_x}{2}$, $yx'<\frac{\rho_{x'}}{2}$ and so

$$xx'\leqq xy+yx'<\frac{\rho_x}{2}+\frac{\rho_{x'}}{2}\leqq \text{max. } (\rho_x,\ \rho_{x'}).$$

This inequality contradicts the definition of ρ_x.

Now let A^x $(x\varepsilon M)$ be a set as follows:

$$\begin{cases} x=\lim\limits_{n\to\infty} x_n,\ x_n\varepsilon E, \\ A^x=\{x_1,\ x_2,\ldots,\ x_n,\ldots\}[3] \subseteq U_x\left(\frac{\rho_x}{5}\right), \\ A^x_3=\{x\}. \end{cases}$$

(The existence of A^x follows from the fact that $x\varepsilon E_\beta$.)

Then, it is clear that

$$A^xA^{x'}=0 \quad \text{for } x\neq x' \ (x,\ x'\varepsilon M)$$

and we may prove that

$$A_3=M_\alpha$$

where

$$A=\sum_{x\varepsilon M} A^x.$$

---

1. E_β means the derived set of E.
2. M_α means the closure of the set M.
3. $\{x_1, x_2,\ldots, x_n\ldots\}$ means the set which consists of the points $x_1, x_2,\ldots, x_n\ldots$.

Since $x\varepsilon A_\beta^x \equiv A_\beta$ for any point of M, we have

(1) $$M_\alpha \equiv A_{\beta\alpha} = A_\beta.$$

On the other hand, for any point y of A_β and for any positive number $\rho > 0$, there is, since $U_y(\rho)A \supset 0$, at least one point $x\varepsilon M$ such that $U_y(\rho)A^x \supset 0$.

If there exist a point $x\varepsilon M$ such as

$$U_y(\rho)A^x \supset 0, \quad \frac{\rho_x}{5} \geqq \rho$$

for a suitable $\rho > 0$, we have

$$xy \leqq xz + zy < \frac{\rho_x}{5} + \rho \leqq \frac{2}{5}\rho_x$$

where $z\varepsilon U_y(\rho)A^x$. While, for a point $\xi\varepsilon (A - A^x)$, there exists a point $x'\varepsilon M$ such that $\xi\varepsilon U_{x'}\left(\frac{\rho_{x'}}{5}\right)$, $x \neq x'$. Hence it follows that

$$y\xi \geqq x\xi - xy \geqq \frac{\rho_x}{2} - \frac{2}{5}\rho_x = \frac{\rho_x}{10} \quad (>0),$$

for $\xi\bar{\varepsilon} U_x\left(\frac{\rho_x}{2}\right)$. In this inequality, ρ_x being independent of ξ, we have

$$y\bar{\varepsilon} (A - A^x)_\beta.$$

And so $y\varepsilon A_\beta^x$ for $y\varepsilon A_\beta$, $A_\beta = A_\beta^x \dot{+} (A - A^x)_\beta$. Therefore, $y\varepsilon M$ for $A_\beta^x = \{x\}$.

If there be no point $x\varepsilon M$ such that

$$A^x U_y(\rho) \supset 0, \quad \frac{\rho_x}{5} \geqq \rho$$

for any positive $\rho > 0$, there exists at least one point $x\varepsilon M$ such that $U_y\left(\frac{1}{n}\right) A^x \supset 0$, $(n = 1, 2, 3, \ldots)$ for $y\varepsilon A_\beta$. And so, for a point $x_{n,y}$ among them and for a point $z_n\varepsilon U_y\left(\frac{1}{n}\right)A^{x_{n,y}}$, we have

$$yx_{n,y} \leqq yz_n + z_n x_{n,y} < \frac{1}{n} + \frac{\rho_{x_{n,y}}}{5} < \frac{2}{n}$$

$$\therefore\ y = \lim_{n\to\infty} x_{n,y}$$

$$\therefore\ y\varepsilon M_\alpha \quad [\because\ x_{n,y}\varepsilon M].$$

Therefore we have $y\varepsilon M_\alpha$ for all points y of A_β and so

(2) $$A_\beta \equiv M_\alpha.$$

The inequalities (1) and (2) prove the theorem.

" *Let E be a metric space. The necessary and sufficient conditions that a set H shall be equal to a derived set of some suitable set, are*

that it is closed in E and does not contain any isolated points of space E."

It is clear that these conditions are necessary. We will show that they are also sufficient.

Let F be a set which satisfies the conditions of this theorem, we have $F \equiv E_\beta$ [$\because$ $FE_j = 0$, $E = E_h + E_j$,[1] $E_h = E_\beta E = E_\beta$].

Now

$$F = F_k + F_s,^{2} \quad F_j \equiv F_s \equiv F_{j\alpha},$$

and from the result which we have had, it is proved that there exists a set A such that

$$F_{j\alpha} = A_\beta.$$

for F_j is an isolated set. Hence it follows that

$$F = (A \dot{+} F_k)_\beta.$$

For, since $(A \dot{+} F_k)_\beta = A_\beta \dot{+} F_{k\beta}$, $F_k \equiv F_{k\beta}$, we have

$$(A \dot{+} F_k)_\beta = A_\beta \dot{+} F_{k\beta} \equiv F_{j\alpha} \dot{+} F_k \equiv F_s + F_k = F.$$

On the other hand,

$$(A \dot{+} F_k)_\beta = A_\beta \dot{+} F_{k\beta} \equiv F,$$

for $A_\beta = F_{j\alpha} \equiv F_\alpha = F$, $F_{k\beta} \equiv F_\beta \equiv F$.

Therefore we have

$$(A \dot{+} F_k)_\beta = F.$$

In conclusion the author wishes to express his sincere thanks to Professor T. Matsumoto for his kind guidance and remarks.

1. $E_h = EE_\beta$, and E_j means the set of isolated points of E.
2. F_k means the nucleus of F, and F_s means the complement of F_k with respect to F.

A Study of the Effect of a Travelling Atmospheric Disturbance upon the Sea-Surface

By Toshichiro Takegami

(Received May 5, 1938)

Abstract

In this paper the writer reports a theoretical study of the fluctuation of sea-surface generated by a pressure disturbance of travelling cyclone with consideration of the effect of a coast. In our investigation, by taking into account not only the forced wave but also the free wave, we have obtained results which may explain tolerably well observed phenomena on the occasion of Kaze-Tunami (the abnormal high water caused by a severe cyclone), and also our solutions thus obtained may be physically reasonable, because they do not give the infinite elevation even when the travelling velocity of the disturbance coincides with the free long wave velocity, resulting in the so-called resonance. And thus we know that our solutions support Nomitsu's opinion.[1]

Moreover, by an example, we have pointed out the possibility that the general solutions can be obtained even if the pressure disturbance varies with the time.

Introduction

Formerly, from simple mathematical calculations, the writer has investigated the elevation or depression of sea-surface generated by a travelling local gale of a large scale (such as a cyclone), and has published his conclusions in this memoir[2]. In this paper we shall investigate the effect of travelling pressure disturbance on the sea-surface and shall complement the results hitherto obtained by further investigations of the same problem.

Many writers have reported interesting results of their investigations of the effect of travelling pressure disturbance on the sea-surface. Especially Dr. Proudman[3] is a pioneer in the study of this problem and he has given from mathematical calculations the theoretical elevation of sea-surface in the following cases: 1) when a pressure disturbance which is independent of time suddenly generates on a sea-surface of unbounded ocean and travels with a constant velocity in one direction; and 2) when, on the ocean bounded on one side by a

1. Mem. Coll. Sci. Kyoto Imp. Univ. A, Vol. **18** (1935), p. 202, 214.
2. Mem. Coll. Sci. Kyoto Imp. Univ. A, Vol. **19** (1936), p. 109.
3. Month. N. Roy. Ast. Soc.. Geophys. Suppl. **2** (1929), p. 197.

coast, it travels towards the coast. But as has been pointed out by Prof. Nomitsu[1], his solutions of the second case take into consideration only the condition at the coast and disregard the initial condition. It seems probable that he has only taken into account the forced wave and has neglected the generated free wave under the assumption that the free wave, owing to the eddy viscosity, dies away before it reaches the coast. But we can not approve these interpretations, and in Prof. Nomitsu's opinion, between land and the open ocean is a continental shelf, whose depth and width are both small as compared with those of the ocean; therefore the free wave generated on the ocean by the pressure disturbance may die away owing to the eddy viscosity before it reaches the continental shelf, but after the disturbance has reached the continental shelf a different free wave will be again generated on account of the sudden variation of depth. And this free wave, even if it is somewhat dampened by friction, will propagate towards the coast with a tolerable height, because, owing to its large propagating velocity and the narrow width of the continental shelf, the time interval in which the free wave propagates from the fore-end of the continental shelf to the coast is short. To explain the surface elevation at the coast and over the continental shelf, therefore, it seems that not only the forced wave but also the free wave must be considered.

Moreover, Proudman's solutions become physically unreasonable when the travelling velocity of disturbance coincides with the long wave velocity and results in the resonance phenomenon, because in such a case his solutions give the infinite elevation of sea-surface. But it has been pointed out first by Prof. Nomitsu[2] and afterwards also by Mr. H. Yamada[3] that this physical inconsistency can be removed by consideration of the eddy viscosity.

In this paper the writer, starting from the initial state of rest and assuming the water to be non-viscous, treats the same problem with consideration of not only the forced wave but also the free wave and explains that low water sometimes occurs before the abnormal high water on the occasion of Kaze-Tunami. And from the same standpoint we give the solutions which give the heights of high and low water caused by a pressure disturbance, and also show that our solu-

1. *l. c. ante.* p. 55.
2. Mem. Coll. Sci. Kyoto Imp. Univ., A, Vol. 18 (1935), p. 213.
3. 海ト空, 第16卷第11號 (1935).

tions do not contain the physical inconsistency noted above even if the so-called resonance phenomenon occurs. Lastly by using Nomitsu's extension[1] of the Duhamel theorem we show, by an example, the possibility that these solutions are in general applicable even if the pressure disturbance varies with the time.

I. Unbounded Sea.

The present problem is treated in two dimensions without consideration of the effect of the earth's rotation and assuming that the sea-water is incompressible and non-viscous—Prof. Nomitsu has treated this problem on an unbounded sea with consideration of the eddy viscosity, but the effect of a coast makes the problem very complicated so that we disregard the eddy viscosity in this paper—and the density of water is taken as unity.

Then using the usual notations, the eq. of motion for the long wave, the eq. of continuity and the initial condition are given as follows :

$$\left.\begin{aligned} &\frac{\partial u}{\partial t}=-g\frac{\partial \zeta}{\partial x}-g\frac{\partial P}{\partial x},\\ &h\frac{\partial u}{\partial x}=-\frac{\partial \zeta}{\partial t}. \end{aligned}\right\} \qquad \cdots\cdots(1)$$

$$\left.\begin{aligned} &\frac{\partial u}{\partial t}=-g\frac{\partial P}{\partial x} \quad \text{or} \quad \frac{\partial \zeta}{\partial t}=0 \quad \text{when } t=0,\\ &u=0 \text{ or } \zeta=0 \qquad\qquad\quad \text{when } t=0. \end{aligned}\right\} \qquad \cdots\cdots(2)$$

Since it is clear that u and ζ are "additive" as defined by Prof. Nomitsu, from Fourier integral and Nomitsu's extension of the Duhamel theorem, the solutions which satisfy (1) and (2) are easily obtained.

$$\frac{h}{c^2}u=-\frac{1}{\pi}\int_0^t d\tau\int_0^\infty d\alpha\int_{-\infty}^\infty \frac{\partial P}{\partial \lambda}\cos\alpha c(t-\tau)\cos\alpha(\lambda-x)d\lambda, \qquad \cdots\cdots(3)$$

$$\zeta=\frac{c}{\pi}\int_0^t d\tau\int_0^\infty d\alpha\int_{-\infty}^\infty \frac{\partial P}{\partial \lambda}\sin\alpha c(t-\tau)\sin\alpha(\lambda-x)d\lambda \qquad \cdots\cdots(4)$$

where $c\equiv gh$ and $P\equiv P(x,\ t)$.

Now, assume that initially ($t=0$) the pressure disturbance whose geometrical centre is $x=0$, suddenly originates on an unbounded sea and travels towards $x<0$ with a constant velocity v. In such a case the pressure disturbance may be given by the form $P(x+vt,\ t)$, then (3) and (4) become

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1. Proc. Imp. Acad. Tokyo, **11**, 359 (1935).

$$\frac{h}{c^2}u=-\frac{1}{2}\int_0^t P'(x-ct+\overline{c+v}\tau,\ \tau)d\tau$$
$$-\frac{1}{2}\int_0^t P'(x+ct-\overline{c-v}\tau,\ \tau)d\tau,\ \ldots\ldots\ldots\ldots(5)$$
$$\zeta=-\frac{c}{2}\int_0^t P'(x-ct+\overline{c+v}\tau,\ \tau)d\tau$$
$$+\frac{c}{2}\int_0^t P'(x+ct-\overline{c-v}\tau,\ \tau)d\tau.\ \ldots\ldots\ldots\ldots(6)$$

where P's are respectively the differentiations with respect to $x-ct+\overline{c+v}\tau$ and $x+ct-\overline{c-v}\tau$. Especially when the travelling velocity v of the disturbance coincides with the long wave velocity c and therefore the resonance phenomenon occurs, (5) and (6) become as follows:

$$\frac{h}{c^2}u=-\frac{1}{2}\int_0^t P'(x+ct,\ \tau)d\tau$$
$$-\frac{1}{2}\int_0^t P'(x-ct+2c\tau,\ \tau)d\tau,\ \ldots\ldots\ldots\ldots(5')$$
$$\zeta=\frac{c}{2}\int_0^t P'(x+ct,\ \tau)d\tau$$
$$-\frac{c}{2}\int_0^t P'(x-ct+2c\tau,\ \tau)d\tau.\ \ldots\ldots\ldots\ldots(6')$$

Now, assume $P=P(x+vt)$, namely, that the shape of the pressure disturbance is unchanged with t, then

$$\frac{h}{c^2}u=\frac{v}{c^2-v^2}P(x+vt)-\frac{1}{2(c-v)}P(x+ct)$$
$$+\frac{1}{2(c+v)}P(x-ct),\ \ldots\ldots\ldots\ldots(5'')$$
$$\zeta=-\frac{c^2}{c^2-v^2}P(x+vt)+\frac{c}{2(c-v)}P(x+ct)$$
$$+\frac{c}{2(c+v)}P(x-ct).\ \ldots\ldots\ldots\ldots(6'')$$

and if $c=v$

$$\frac{h}{c^2}u=-\frac{t}{2}P'(x+ct)-\frac{1}{4c}P(x+ct)$$
$$+\frac{1}{4c}P(x-ct),\ \ldots\ldots\ldots\ldots(5''')$$
$$\zeta=\frac{ct}{2}P'(x+ct)-\frac{1}{4}P(x+ct)+\frac{1}{4}P(x-ct).\ \ldots(6''')$$

Eqs. (5'') and (6'') are the same as Proudman's solutions. And Dr. Proudman gives solutions resembling (5), (6), (5') and (6'), but

his solutions contain only the effect of time variation of disturbance and not its travelling effect, while our solutions contain both factors, and therefore may be more general than Proudman's solutions. Moreover from these (5), (6), (5′) and (6′) we may obtain solutions taking into consideration the time variation of disturbance, the effect of a coast and the travelling effect, but these will be stated in detail in the last section.

II. Sea Bounded by a Straight Coast.

In this case a boundary condition

$$u=0 \quad \text{or} \quad \frac{\partial\zeta}{\partial x}=-\frac{\partial P}{\partial x} \qquad \text{at } x=0 \qquad \cdots\cdots(2')$$

must be added to (2).

Now, let the ocean be in $x>0$, and the land be in $x<0$. If we take as the time origin an instant when the fore-end of disturbance arrives at the coast and consider only the forced wave without the free wave, then we have

$$\left.\begin{aligned}\frac{h}{c^2}u&=\frac{v'}{c^2-v'^2}\left\{P\left(t+\frac{x}{v'}\right)-P\left(t-\frac{x}{c}\right)\right\},\\ \zeta&=-\frac{c^2}{c^2-v'^2}\left\{P\left(t+\frac{x}{v'}\right)+\frac{v'}{c}P\left(t-\frac{x}{c}\right)\right\},\end{aligned}\right\}\cdots\cdots(7)$$

These are the original forms first given by Dr. Proudman and later by Mr. H. Yamada. The first term is the incident forced wave and the second term is the reflected wave, but as pointed out by Prof. Nomitsu these solutions do not satisfy the initial condition of rest.

Now assume

$$P(\xi)=P(x+vt-a) \qquad \text{for } -l<\xi<+l,$$
$$P(\xi)=0 \qquad \text{for } \xi<-l \text{ and } \xi>+l.$$

where a is the co-ordinate of the geometrical centre of disturbance at $t=0$ and $2l$ is the width of disturbance. Then the solutions corresponding to (7) are as follows, but the argument of P is suitably replaced because the free wave is considered.

$$\left.\begin{aligned}\frac{h}{c^2}u&=\frac{v'}{c^2-v'^2}\left\{P(x+vt-a)-P\left(vt-a-\frac{v'}{c}x\right)\right\},\\ \zeta&=-\frac{c^2}{c^2-v'^2}\left\{P(x+vt-a)+\frac{v'}{c}P\left(vt-a-\frac{v'}{c}x\right)\right\},\end{aligned}\right\}\cdots\cdots(7')$$

From the suitable combination of (7′), (5″) and (6″) we shall obtain the solutions which satisfy both the initial and the boundary conditions.

Now, divide the time interval into five or three stages until the disturbance has passed into the land, then the solution for the respective stages will be as follows. Here $c>v$ is assumed.

1) $0\leq t\leq\frac{a-l}{c}$: *Before the pressure disturbance and the generated free wave arrive at the coast.*

$$\frac{h}{c^2}u=\frac{v}{c^2-v^2}P(x+vt-a)-\frac{1}{2(c-v)}P(x+ct-a)+\frac{1}{2(c+v)}P(x-ct-a),$$

$$\zeta=-\frac{c^2}{c^2-v^2}P(x+vt-a)+\frac{c}{2(c-v)}P(x+ct-a)+\frac{c}{2(c+v)}P(x-ct-a).$$

It is obvious that these solutions satisfy (1) and (2); and since in this stage the wave has not arrived at the coast, the boundary condition (2′) is also satisfied.

2) $\frac{a-l}{c}\leq t\leq\frac{a+l}{c}$: *After the free wave has arrived but before the pressure disturbance arrives and the free wave is completely reflected.*

$$\frac{h}{c^2}u=\frac{v}{c^2-v^2}P(x+vt-a)-\frac{1}{2(c-v)}\{P(x+ct-a)-P(ct-a-x)\}+\frac{1}{2(c+v)}P(x-ct-a),$$

$$\zeta=-\frac{c^2}{c^2-v^2}P(x+vt-a)+\frac{c}{2(c-v)}\{P(x+ct-a)+P(ct-a-x)\}+\frac{c}{2(c+v)}P(x-ct-a).$$

In this stage it is not necessary that the initial condition is satisfied. The first and the third term of u have no contribution at the coast from the character of P and the second term of u obviously vanishes at the coast ($x=0$), therefore the boundary condition (2′) is satisfied. Substituting these u and ζ in (1), it is easily verified that they satisfy eq. (1). Moreover u and ζ change continuously from stage 1) to stage 2), because, when $t=\frac{a-l}{c}$, the term $P(ct-a-x)$ becomes $P(-l-x)$ which vanishes on the sea everywhere. Similarly we see that the solutions of following stages (3), (4), (5), (3′), (4′) and (5′) satisfy eq. (1), the boundary condition at the coast (2′) and

the continuity of the transition from one stage to another.

3) $\frac{a+l}{c} \leq t \leq \frac{a-l}{v'}$: *After the free wave has been reflected completely but before the pressure disturbance arrives.*

$$\frac{h}{c^2}u = \frac{v'}{c^2-v'^2}P(x+v't-a) + \frac{1}{2(c-v')}P(ct-a-x) + \frac{1}{2(c+v')}P(x-ct-a),$$

$$\zeta = -\frac{c^2}{c^2-v'^2}P(x+v't-a) + \frac{c}{2(c-v')}P(ct-a-x) + \frac{c}{2(c+v')}P(x-ct-a).$$

4) $\frac{a-l}{v'} \leq t \leq \frac{a+l}{v'}$: *After the pressure disturbance has arrived but before it has completely passed away inland.*

$$\frac{h}{c^2}u = \frac{v'}{c^2-v^2}\left\{P(x+vt-a) - P\left(v't-a-\frac{v'}{c}x\right)\right\} + \frac{1}{2(c-v')}P(ct-a-x) + \frac{1}{2(c+v')}P(x-ct-a),$$

$$\zeta = -\frac{c^2}{c^2-v'^2}\left\{P(x+v't-a) + \frac{v'}{c}P\left(v't-a-\frac{v'}{c}x\right)\right\} + \frac{c}{2(c-v')}P(ct-a-x) + \frac{c}{2(c+v')}P(x-ct-a).$$

5) $\frac{a+l}{v'} \leq t$: *After the pressure disturbance has completely passed away inland.*

$$\frac{h}{c^2}u = -\frac{v}{c^2-v'^2}P\left(v't-a-\frac{v'}{c}x\right) + \frac{1}{2(c-v')}P(ct-a-x) + \frac{1}{2(c+v')}P(x-ct-a),$$

$$\zeta = -\frac{cv'}{c^2-v'^2}P\left(v't-a-\frac{v'}{c}x\right) + \frac{c}{2(c-v')}P(ct-a-x) + \frac{c}{2(c+v')}P(x-ct-a).$$

Moreover, due to the relation of the relative magnitude of c and v', another case may be considered as follows:

3′) $\frac{a-l}{c} < \frac{a-l}{v'} \leq t \leq \frac{a+l}{c}$: *After the pressure disturbance has arrived but before the free wave has been reflected completely.*

$$\frac{h}{c^2}u=\frac{v'}{c^2-v'^2}\left\{P(x+v't-a)-P\left(v't-a-\frac{v'}{c}x\right)\right\}$$
$$-\frac{1}{2(c-v')}\{P(x+ct-a)-P(ct-a-x)\}$$
$$+\frac{1}{2(c+v')}P(x-ct-a),$$
$$\zeta=-\frac{c^2}{c^2-v'^2}\left\{P(x+v't-a)+\frac{v'}{c}P\left(v't-a-\frac{v'}{c}x\right)\right\}$$
$$+\frac{c}{2(c-v')}\{P(x+ct-a)+P(ct-a-x)\}$$
$$+\frac{c}{2(c+v')}P(x-ct-a).$$

4′) $\frac{a-l}{v'}<\frac{a+l}{c}\leq t\leq\frac{a+l}{v'}$: *After the free wave has been reflected but before the pressure disturbance has completely passed away inland.*

5′) $\frac{a+l}{v'}\leq t$: *After the pressure disturbance has completely passed away inland.*

The solutions of these stages 4′) and 5′) are the same as those of 4) and 5).

The solutions for the case of $c<v'$ have only the difference that the pressure disturbance arrives earlier than the free wave, and contain no other essential difference from the above forms. Therefore we will not state them in detail. But when $c=v'$ and the resonance occurs, starting from (5‴) and (6‴) the solutions corresponding to the above forms may be obtained as follows:

1) $0\leq t\leq\frac{a-l}{c}$: *Before the pressure disturbance arrives at the coast.*

$$\frac{h}{c^2}u=-\frac{t}{2}P'(x+ct-a)-\frac{1}{4c}P(x+ct-a)$$
$$+\frac{1}{4c}P(x-ct-a),$$
$$\zeta=\frac{ct}{2}P'(x+ct-a)-\frac{1}{4}P(x+ct-a)+\frac{1}{4}P(x-ct-a).$$

2) $\frac{a-l}{c}\leq t\leq\frac{a+l}{c}$: *After the pressure disturbance has arrived but before it has completely passed away inland.*

$$\frac{h}{c^2}u=-\frac{t}{2}P'(x+ct-a)+\frac{1}{2}\left(t-\frac{x}{c}\right)P'(ct-a-x)$$

$$-\frac{1}{4c}P(x+ct-a)+\frac{1}{4c}P(ct-a-x)$$
$$+\frac{1}{4c}P(x-ct-a),$$
$$\zeta=\frac{ct}{2}P'(x+ct-a)+\frac{c}{2}\left(t-\frac{x}{c}\right)P'(ct-a-x)$$
$$-\frac{1}{4}P(x+ct-a)+\frac{1}{4}P(ct-a-x)$$
$$+\frac{1}{4}P(x-ct-a).$$

3) $\frac{a+l}{c}\leq t$: *After the pressure disturbance has completely passed away inland.*

$$\frac{h}{c^2}u=\frac{1}{2}\left(t-\frac{x}{c}\right)P'(ct-a-x)+\frac{1}{4c}P(ct-a-x)$$
$$+\frac{1}{4c}P(x-ct-a),$$
$$\zeta=\frac{c}{2}\left(t-\frac{x}{c}\right)P'(ct-a-x)+\frac{1}{4}P(ct-a-x)$$
$$+\frac{1}{4}P(x-ct-a).$$

In these formulae it may be noticed that the height of the forced wave is proportional to the time interval after the disturbance occurred, and therefore it does not increase to become infinitely large so long as the time interval is finite. Of course these facts emerged from consideration of the initial condition, but it has already been shown by Prof. Nomitsu that the physical inconsistency in the resonance case—the height of wave increases infinitely if $c=v$—may be removed by taking friction into account even if the time interval is infinite.

When we wish to apply our formulae to a practical case, we must have the actual meteorological record of atmospheric pressure P, the presumed travelling velocity of cyclone v, the mean depth of continental shelf h, the width of disturbance $2l$ and the initial position of the centre of disturbance a (a may be assumed as the sum of the width of the continental shelf and half the width of disturbance, l). If these date are known, the dynamic elevation of the sea-surface may be easily calculated. But it must be noticed that, since our solutions are obtained on the assumption of the uniformity of depth, the direct application of our formulae to the boundary between the continental shelf and the ocean may not be rigorously exact. In order to obtain

a more reasonable solution which may be applied at the boundary, we must divide the sea-surface into two parts; namely, an ocean and a continental shelf, and suitably combine the solutions which hold in the respective parts and their boundary. But this treatment will make the problem more complicated; therefore we do not touch on it in this paper but shall leave it for another occasion.

Now, for an example, assume that the real pressure record is given in the following form:

$$\left.\begin{aligned} P&=-A\left\{1+\cos\frac{\pi}{l}(x+v't-a)\right\} && \text{for} \quad a-l<x+v't<a+l,\\ P&=0 && \text{for} \quad x+v't<a-l \quad \text{and} \\ & && \qquad x+v't>a+l. \end{aligned}\right\} \qquad \cdots\cdots(8)$$

1) $0\leq t\leq\dfrac{a-l}{c}$:

$$\begin{aligned}\frac{h}{c^2}u=&-\frac{v'}{c^2-v'^2}A\left\{1+\cos\frac{\pi}{l}(x+v't-a)\right\}\\&+\frac{1}{2(c-v')}A\left\{1+\cos\frac{\pi}{l}(x+ct-a)\right\}\\&-\frac{1}{2(c+v')}A\left\{1+\cos\frac{\pi}{l}(x-ct-a)\right\},\end{aligned}$$

$$\begin{aligned}\zeta=&\frac{c^2}{c^2-v'^2}A\left\{1+\cos\frac{\pi}{l}(x+v't-a)\right\}\\&-\frac{c}{2(c-v')}A\left\{1+\cos\frac{\pi}{l}(x+ct-a)\right\}\\&-\frac{c}{2(c+v')}A\left\{1+\cos\frac{\pi}{l}(x-ct-a)\right\}.\end{aligned}$$

2) $\dfrac{a-l}{c}\leq t\leq\dfrac{a+l}{c}<\dfrac{a-l}{v'}$:

$$\begin{aligned}\frac{h}{c^2}u=&-\frac{v'}{c^2-v'^2}A\left\{1+\cos\frac{\pi}{l}(x+v't-a)\right\}\\&+\frac{1}{2(c-v')}A\left\{\left[1+\cos\frac{\pi}{l}(x+ct-a)\right]\right.\\&\left.-\left[1+\cos\frac{\pi}{l}(ct-a-x)\right]\right\}\\&-\frac{1}{2(c+v')}A\left\{1+\cos\frac{\pi}{l}(x-ct-a)\right\},\end{aligned}$$

$$\zeta=\frac{c^2}{c^2-v'^2}A\left\{1+\cos\frac{\pi}{l}(x+v't-a)\right\}$$

$$-\frac{c}{2(c-v')}A\left\{\left[1+\cos\frac{\pi}{l}(x+ct-a)\right]\right.$$
$$\left.+\left[1+\cos\frac{\pi}{l}(ct-a-x)\right]\right\}$$
$$-\frac{c}{2(c+v')}A\left\{1+\cos\frac{\pi}{l}(x-ct-a)\right\}.$$

3) $\frac{a+l}{c}\leq t\leq\frac{a-l}{v'}$:

$$\frac{h}{c^2}w=-\frac{v'}{c^2-v'^2}A\left\{1+\cos\frac{\pi}{l}(x+v't-a)\right\}$$
$$-\frac{1}{2(c-v')}A\left\{1+\cos\frac{\pi}{l}(ct-a-x)\right\}$$
$$-\frac{1}{2(c+v')}A\left\{1+\cos\frac{\pi}{l}(x-ct-a)\right\},$$
$$\zeta=\frac{c^2}{c^2-v'^2}A\left\{1+\cos\frac{\pi}{l}(x+v't-a)\right\}$$
$$-\frac{c}{2(c-v')}A\left\{1+\cos\frac{\pi}{l}(ct-a-x)\right\}$$
$$-\frac{c}{2(c+v')}A\left\{1+\cos\frac{\pi}{l}(x-ct-a)\right\}.$$

4) $\frac{a-l}{v'}\leq t\leq\frac{a+l}{v'}$:

$$\frac{h}{c^2}w=-\frac{v'}{c^2-v'^2}A\left\{\left[1+\cos\frac{\pi}{l}(x+v't-a)\right]\right.$$
$$\left.-\left[1+\cos\frac{\pi}{l}\left(v't-a-\frac{v'}{c}x\right)\right]\right\}$$
$$-\frac{1}{2(c-v')}A\left\{1+\cos\frac{\pi}{l}(ct-a-x)\right\}$$
$$-\frac{1}{2(c+v')}A\left\{1+\cos\frac{\pi}{l}(x-ct-a)\right\},$$
$$\zeta=\frac{c^2}{c^2-v'^2}A\left\{\left[1+\cos\frac{\pi}{l}(x+v't-a)\right]\right.$$
$$\left.+\frac{v'}{c}\left[1+\cos\frac{\pi}{l}\left(v't-a-\frac{v'}{c}x\right)\right]\right\}$$
$$-\frac{c}{2(c-v')}A\left\{1+\cos\frac{\pi}{l}(ct-a-x)\right\}$$
$$-\frac{c}{2(c+v')}A\left\{1+\cos\frac{\pi}{l}(x-ct-a)\right\}.$$

5) $\dfrac{a+l}{v'}\leq t$:

$$\frac{h}{c^2}u=\frac{v'}{c^2-v'^2}A\left\{1+\cos\frac{\pi}{l}\left(v't-a-\frac{v'}{c}x\right)\right\}$$
$$-\frac{1}{2(c-v')}A\left\{1+\cos\frac{\pi}{l}(ct-a-x)\right\}$$
$$-\frac{1}{2(c+v')}A\left\{1+\cos\frac{\pi}{l}(x-ct-a)\right\},$$
$$\zeta=\frac{v'c}{c^2-v'^2}A\left\{1+\cos\frac{\pi}{l}\left(v't-a-\frac{v'}{c}x\right)\right\}$$
$$-\frac{c}{2(c-v')}A\left\{1+\cos\frac{\pi}{l}(ct-a-x)\right\}$$
$$-\frac{c}{2(c+v')}A\left\{1+\cos\frac{\pi}{l}(x-ct-a)\right\}.$$

3') $\dfrac{a-l}{c}<\dfrac{a-l}{v'}\leq t\leq\dfrac{a+l}{c}$:

$$\frac{h}{c^2}u=-\frac{v'}{c^2-v'^2}A\left\{\left[1+\cos\frac{\pi}{l}(x+v't-a)\right]\right.$$
$$\left.-\left[1+\cos\frac{\pi}{l}\left(v't-a-\frac{v'}{c}x\right)\right]\right\}$$
$$+\frac{1}{2(c-v')}A\left\{\left[1+\cos\frac{\pi}{l}(x+ct-a)\right]\right.$$
$$\left.-\left[1+\cos\frac{\pi}{l}(ct-a-x)\right]\right\}$$
$$-\frac{1}{2(c+v')}A\left\{1+\cos\frac{\pi}{l}(x-ct-a)\right\},$$
$$\zeta=\frac{c^2}{c^2-v'^2}A\left\{\left[1+\cos\frac{\pi}{l}(x+v't-a)\right]\right.$$
$$\left.+\frac{v'}{c}\left[1+\cos\frac{\pi}{l}\left(v't-a-\frac{v'}{c}x\right)\right]\right\}$$
$$-\frac{c}{2(c-v')}A\left\{\left[1+\cos\frac{\pi}{l}(x+ct-a)\right]\right.$$
$$\left.+\left[1+\cos\frac{\pi}{l}(ct-a-x)\right]\right\}$$
$$-\frac{c}{2(c+v')}A\left\{1+\cos\frac{\pi}{l}(x-ct-a)\right\}.$$

4') $\dfrac{a-l}{v'}<\dfrac{a+l}{c}\leq t\leq\dfrac{a+l}{v}$.

5') $\dfrac{a+l}{v'}\leq t$.

The solutions of stages 4′) and 5′) are respectively the same as those of 4) and 5).

If $v=c$, we have

1) $0\leq t\leq \frac{a-l}{c}$:

$$\frac{h}{c^2}u=-\frac{\pi t}{2l}A\sin\frac{\pi}{l}(x+ct-a)+\frac{1}{4c}A\left\{1+\cos\frac{\pi}{l}(x+ct-a)\right\}-\frac{1}{4c}A\left\{1+\cos\frac{\pi}{l}(x-ct-a)\right\},$$

$$\zeta=\frac{\pi ct}{2l}A\sin\frac{\pi}{l}(x+ct-a)+\frac{1}{4}A\left\{1+\cos\frac{\pi}{l}(x+ct-a)\right\}-\frac{1}{4}A\left\{1+\cos\frac{\pi}{l}(x-ct-a)\right\}.$$

2) $\frac{a-l}{c}\leq t\leq\frac{a+l}{c}$:

$$\frac{h}{c^2}u=-\frac{\pi t}{2l}A\sin\frac{\pi}{l}(x+ct-a)+\frac{\pi}{2l}\left(t-\frac{x}{c}\right)\sin\frac{\pi}{l}(ct-a-x)+\frac{1}{4c}A\left\{\left[1+\cos\frac{\pi}{l}(x+ct-a)\right]-\left[1+\cos\frac{\pi}{l}(ct-a-x)\right]\right\}-\frac{1}{4c}A\left\{1+\cos\frac{\pi}{l}(x-ct-a)\right\},$$

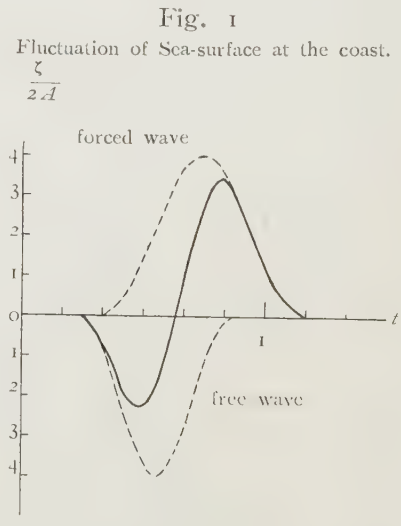
$$\zeta=\frac{\pi ct}{2l}A\sin\frac{\pi}{l}(x+ct-a)+\frac{\pi c}{2l}\left(t-\frac{x}{c}\right)A\sin\frac{\pi}{l}(ct-a-x)+\frac{1}{4}A\left\{\left[1+\cos\frac{\pi}{l}(x+ct-a)\right]-\left[1+\cos\frac{\pi}{l}(ct-a-x)\right]\right\}-\frac{1}{4}A\left\{1+\cos\frac{\pi}{l}(x-ct-a)\right\}.$$

3) $\frac{a+l}{c} \leq t$:

$$\frac{h}{c^2}u = \frac{\pi}{2l}\left(t-\frac{x}{c}\right)\cdot A\sin\frac{\pi}{l}(ct-a-x)$$
$$-\frac{1}{4c}\cdot A\left\{1+\cos\frac{\pi}{l}(ct-a-x)\right\}$$
$$-\frac{1}{4c}A\left\{1+\cos\frac{\pi}{l}(x-ct-a)\right\},$$
$$\zeta = \frac{\pi c}{2l}\left(t-\frac{x}{c}\right)\cdot A\sin\frac{\pi}{l}(ct-a-x)$$
$$-\frac{1}{4}A\left\{1+\cos\frac{\pi}{l}(ct-a-x)\right\}$$
$$-\frac{1}{4}A\left\{1+\cos\frac{\pi}{l}(x-ct-a)\right\}.$$

Again for an example, let us assume that the width of continental shelf is 20 km, its mean depth, h, is 50 m ($\therefore$ $c=80$ km/hour) and the width of the pressure disturbance $2l$ is 50 km; then the distance a is 45 km. And if we consider three cases in which the travelling velocity v takes 60 km/hour, 20 km/hour and 80 km/hour respectively, then the time curves of surface elevation of the above three cases at the coast will be as shown in Fig. 1, 2 and 3, in which t (unit is hour) is abcissa and $\frac{\zeta}{2A}$ ($2A$ is the statical elevation caused by the pressure disturbance) is ordinate.

Fig. 1
Fluctuation of Sea-surface at the coast.



From the above figures it must be noticed that in all the cases low water due to the free wave first arrives at the coast and later high water caused by the forced wave arrives. (If $v>c$ the circumstances are contrary to the above, but $v>c$ scarcely ever happens.) Owing to the eddy viscosity the low water may not be so remarkable as the figures show, but by taking the free wave into account it may be easily explained that the low water sometimes appears at the coast earlier than the high water as a precurser of Kaze-Tunami.

Fig. 2

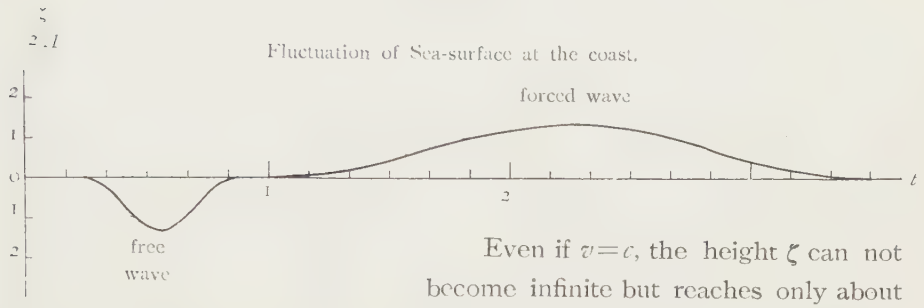
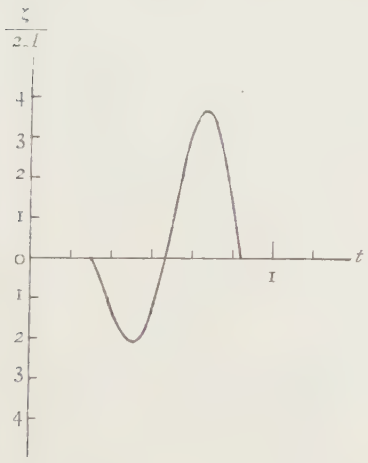


Fig. 3

Fluctuation of Sea-surface at the coast.



Even if $v=c$, the height ζ can not become infinite but reaches only about four-times the statical elevation, namely these circumstances support Nomitsu's[1] opinion which he expressed in reporting his investigation of Kaze-Tunami.

Now, if we assume $2A=(760-682)\times 13.6$ mm $=106$ cm (682 mm is the minimum reading of barometer at Muroto, Sept. 21, 1934), and calculate the minimum and the maximum value of ζ for the above three cases, then we have respectively 1) 1.37 m and 3.6 m, 2) 1.42 m and 1.42 m and 3) 2.22 m and 3.9 m. Formerly the writer has given a formula which gives the height of high water generated by the travelling gale of a large scale. If we take 30 m/sec as the wind velocity, and apply that formula to the present case, then the high waters of the three cases are respectively 42 cm, 32 cm and 42 cm. Thus it seems that the wind effect may not be so great as the pressure effect.

The heights of high water originating from wind and pressure become respectively about 4 m, 1.7 m and 4.3 m.

Since $v=60$ km/hour is considered the most appropriate value for the travelling velocity of the Muroto Typhoon of Sept. 21, 1934, 4 m may be taken as the total calculated height of the high water that attacked the southern coast of Shikoku in Japan. And this value will be sufficient to explain the actual height of water reported in the case of the Kaze-Tunami of Sept. 21, 1934.

1. *l. c. ante.* pp. 55, 56.

III. When the Pressure Disturbance is a Function of Time, for Example, $P=tP(x+vt-a)$

Even when the travelling atmospheric disturbance is a function of time, the solutions for the unbounded sea, (5), (6), (5′) and (6′) can be used. If the effect of coast is considered, the solutions may be also obtained by the suitable combination of the reflected wave with the incident wave corresponding to (5), (6). But since to obtain the solutions for the general form of P is difficult in respect to the integration, we shall show its possibility only with an example in which P is a linear function of t.

1) **Unbounded Sea.**

In this case the disturbance is given by $tP(x+vt)$. Substituting it in (5), (6), (5′) and (6′) we have the following solutions:

$$\frac{h}{c^2}u=-\frac{1}{2}\int_0^t \tau P'(x-ct+\overline{c+v}\tau)d\tau-\frac{1}{2}\int_0^t \tau P'(x+ct-\overline{c-v}\tau)d\tau$$

$$=\frac{v}{c^2-v^2}tP(x+vt)+\frac{c^2+v^2}{(c^2-v^2)^2}\phi(x+vt)-\frac{1}{2(c-v)^2}\phi(x+ct)$$

$$-\frac{1}{2(c+v)^2}\phi(x-ct),$$

$$\zeta=-\frac{c}{2}\int_0^t \tau P'(x-ct+\overline{c+v}\tau)d\tau+\frac{c}{2}\int_0^t \tau P'(x+ct-\overline{c-v}\tau)d\tau$$

$$=-\frac{c^2}{c^2-v^2}tP(x+vt)-\frac{2vc^2}{(c^2-v^2)^2}\phi(x+vt)+\frac{c}{2(c-v)^2}\phi(x+ct)$$

$$-\frac{c}{2(c+v)^2}\phi(x-ct).$$

If $v=c$

$$\frac{h}{c^2}u=-\frac{1}{2}\int_0^t \tau P'(x+ct)d\tau-\frac{1}{2}\int_0^t \tau P'(x-ct+2c\tau)d\tau$$

$$=-\frac{1}{4}t^2P'(x+ct)-\frac{1}{4c}tP(x+ct)$$

$$+\frac{1}{8c^2}\{\phi(x+ct)-\phi(x-ct)\},$$

$$\zeta=\frac{c}{2}\int_0^t \tau P'(x+ct)d\tau-\frac{c}{2}\int_0^t \tau P'(x-ct+2c\tau)d\tau$$

$$=\frac{c}{4}t^2P'(x+ct)-\frac{1}{4}tP(x+ct)$$

$$+\frac{1}{8c}\{\phi(x+ct)-\phi(x-ct)\}.$$

where $\phi'=P$.

2) **Bounded Sea.**

In this case the disturbance is given by $tP(x+vt-a)$. By the suitable combination of the incident wave and the reflected wave, we get the required solutions as follows:

1) $0 \leq t \leq \frac{a-l}{c}$:

$$\frac{h}{c^2}u = \frac{v}{c^2-v^2}tP(x+vt-a) + \frac{c^2+v^2}{(c^2-v^2)^2}\phi(x+vt-a)$$
$$-\frac{1}{2(c-v)^2}\phi(x+ct-a) - \frac{1}{2(c+v)^2}\phi(x-ct-a),$$
$$\zeta = -\frac{c^2}{c^2-v^2}tP(x+vt-a) - \frac{2vc^2}{(c^2-v^2)^2}\phi(x+vt-a)$$
$$+\frac{c}{2(c-v)^2}\phi(x+ct-a) - \frac{c}{2(c+v)^2}\phi(x-ct-a).$$

2) $\frac{a-l}{c} \leq t \leq \frac{a+l}{c}$:

$$\frac{h}{c^2}u = \frac{v}{c^2-v^2}tP(x+vt-a) + \frac{c^2+v^2}{(c^2-v^2)^2}\phi(x+vt-a)$$
$$-\frac{1}{2(c-v)^2}\{\phi(x+ct-a) - \phi(ct-a-x)\}$$
$$-\frac{1}{2(c+v)^2}\phi(x-ct-a),$$
$$\zeta = -\frac{c^2}{c^2-v^2}tP(x+vt-a) - \frac{2vc^2}{(c^2-v^2)^2}\phi(x+vt-a)$$
$$+\frac{c}{2(c-v)^2}\{\phi(x+ct-a) + \phi(ct-a-x)\}$$
$$-\frac{c}{2(c+v)^2}\phi(x-ct-a).$$

3) $\frac{a+l}{c} \leq t \leq \frac{a-l}{v}$:

$$\frac{h}{c^2}u = \frac{v}{c^2-v^2}tP(x+vt-a) + \frac{c^2+v^2}{(c^2-v^2)^2}\phi(x+vt-a)$$
$$+\frac{1}{2(c-v)^2}\phi(ct-a-x) - \frac{1}{2(c+v)^2}\phi(x-ct-a),$$
$$\zeta = -\frac{c^2}{c^2-v^2}tP(x+vt-a) - \frac{2vc^2}{(c^2-v^2)^2}\phi(x+vt-a)$$
$$+\frac{c}{2(c-v)^2}\phi(ct-a-x) - \frac{c}{2(c+v)^2}\phi(x-ct-a).$$

4) $\dfrac{a-l}{v'} \leq t \leq \dfrac{a+l}{v'}$:

$$\frac{h}{c^2}u = \frac{v}{c^2-v^2}\left\{tP(x+vt-a)-\left(t-\frac{x}{c}\right)P\left(vt-a-\frac{v}{c}x\right)\right\}$$
$$+\frac{c^2+v^2}{(c^2-v^2)^2}\left\{\phi(x+vt-a)-\phi\left(vt-a-\frac{v}{c}x\right)\right\}$$
$$+\frac{1}{2(c-v)^2}\phi(ct-a-x)-\frac{1}{2(c+v)^2}\phi(x-ct-a),$$

$$\zeta = -\frac{c^2}{c^2-v^2}\left\{tP(x+vt-a)+\frac{v}{c}\left(t-\frac{x}{c}\right)P\left(vt-a-\frac{v}{c}x\right)\right\}$$
$$-\frac{c^2}{(c^2-v^2)^2}\left\{2v\phi(x+vt-a)+\frac{c^2+v^2}{c}\phi\left(vt-a-\frac{v}{c}x\right)\right\}$$
$$+\frac{c}{2(c-v)^2}\phi(ct-a-x)-\frac{c}{2(c+v)^2}\phi(x-ct-a).$$

5) $\dfrac{a+l}{v'} \leq t$:

$$\frac{h}{c^2}u = -\frac{v}{c^2-v^2}\left(t-\frac{x}{c}\right)P\left(vt-a-\frac{v}{c}x\right)$$
$$-\frac{c^2+v^2}{(c^2-v^2)^2}\phi\left(vt-a-\frac{v}{c}x\right)+\frac{1}{2(c-v)^2}\phi(ct-a-x)$$
$$-\frac{1}{2(c+v)^2}\phi(x-ct-a),$$

$$\zeta = -\frac{vc}{c^2-v^2}\cdot\left(t-\frac{x}{c}\right)P\left(vt-a-\frac{v}{c}x\right)-\frac{c(c^2+v^2)}{(c^2-v^2)^2}\phi\left(vt-a-\frac{v}{c}x\right)$$
$$+\frac{c}{2(c-v)^2}\phi(ct-a-x)-\frac{c}{2(c+v)^2}\phi(x-ct-a).$$

3') $\dfrac{a-l}{c} < \dfrac{a-l}{v} \leq t \leq \dfrac{a+l}{c}$:

$$\frac{h}{c^2}u = \frac{v}{c^2-v^2}\left\{tP(x+vt-a)-\left(t-\frac{x}{c}\right)P\left(vt-a-\frac{v}{c}x\right)\right\}$$
$$+\frac{c^2+v^2}{(c^2-v^2)^2}\left\{\phi(x+vt-a)-\phi\left(vt-a-\frac{v}{c}x\right)\right\}$$
$$-\frac{1}{2(c-v)^2}\{\phi(x+ct-a)-\phi(ct-a-x)\}$$
$$-\frac{1}{2(c+v)^2}\phi(x-ct-a),$$

$$\zeta = -\frac{c^2}{c^2-v^2}\left\{tP(x+vt-a)+\frac{v}{c}\left(t-\frac{x}{c}\right)P\left(vt-a-\frac{v}{c}x\right)\right\}$$

$$-\frac{c^2}{(c^2-v^2)^2}\left\{2v\phi(x+vt-a)+\frac{c^2+v^2}{c}\phi\left(vt-a-\frac{v}{c}x\right)\right\}$$

$$+\frac{c}{2(c-v)^2}\{\phi(x+ct-a)+\phi(ct-a-x)\}-\frac{c}{2(c+v)^2}\phi(x-ct-a).$$

4′) $\quad\dfrac{a-l}{v}<\dfrac{a+l}{c}\leq t\leq\dfrac{a+l}{v}.$

5′) $\quad\dfrac{a+l}{v}\leq t.$

The solutions of the stages 4′) and 5′) are the same as those of 4) and 5).

If $v=c$,

1) $\quad 0\leq t\leq\dfrac{a-l}{c}$:

$$\frac{h}{c^2}u=-\frac{1}{4}t^2P'(x+ct-a)-\frac{1}{4c}tP(x+ct-a)$$

$$+\frac{1}{8c^2}\{\phi(x+ct-a)-\phi(x-ct-a)\},$$

$$\zeta=\frac{c}{4}t^2P'(x+ct-a)-\frac{1}{4}tP(x+ct-a)$$

$$+\frac{1}{8c}\{\phi(x+ct-a)-\phi(x-ct-a).$$

2) $\quad\dfrac{a-l}{c}\leq t\leq\dfrac{a+l}{c}$:

$$\frac{h}{c^2}u=-\frac{1}{4}\left\{t^2P'(x+ct-a)-\left(t-\frac{x}{c}\right)^2P'(ct-a-x)\right\}$$

$$-\frac{1}{4c}\left\{tP(x+ct-a)-\left(t-\frac{x}{c}\right)P(ct-a-x)\right\}$$

$$+\frac{1}{8c^2}\{\phi(x+ct-a)-\phi(ct-a-x)-\phi(x-ct-a)\},$$

$$\zeta=\frac{c}{4}\left\{t^2P'(x+ct-a)+\left(t-\frac{x}{c}\right)^2P'(ct-a-x)\right\}$$

$$-\frac{1}{4}\left\{tP(x+ct-a)-\left(t-\frac{x}{c}\right)P(ct-a-x)\right\}$$

$$+\frac{1}{8c}\{\phi(x+ct-a)-\phi(ct-a-x)-\phi(x-ct-a)\}.$$

3) $\quad\dfrac{a+l}{c}\leq t$:

$$\frac{h}{c^2}u=\frac{1}{4}\left(t-\frac{x}{c}\right)^2P'(ct-a-x)+\frac{1}{4c}\left(t-\frac{x}{c}\right)P(ct-a-x)$$

$$-\frac{1}{8c^2}\{\phi(ct-a-x)+\phi(x-ct-a)\},$$

$$\zeta=\frac{c}{4}\left(t-\frac{x}{c}\right)^2 P'(ct-a-x)+\frac{1}{4}\left(t-\frac{x}{c}\right)P(ct-a-x)$$

$$-\frac{1}{8c}\{\phi(ct-a-x)+\phi(x-ct-a)\}.$$

By the direct substitutions of the above formulae in (1), (2) and (2′) we can verify the required solutions.

If the form of P is more general, we may obtain the solutions by similar combinations of the incident wave and the reflected wave. Here we point out only the solutions of an example and the possibility that the solutions may be given for the general form of P, but give no numerical calculation.

Summary

1) In the unbounded sea the generated free wave may die away in a long time owing to the eddy viscosity. But in a sea bounded by a straight coast, after the disturbance has arrived at the fore-end of the continental shelf, there can be no question that the free wave will be newly generated on account of the sudden variation of depth. Therefore if we wish to treat the problem of the elevation of sea-surface in the continental shelf, we must start, as Prof. Nomitsu has pointed out, from the initial state of rest and consequently take into account the free wave. Thus we have obtained the appropriate solutions in the general form and explained the real phenomena on the occasion of Kaze-Tunami.

2) By considering the initial state, we have the solution which is physically reasonable even if $v=c$.

3) And we have shown, by an example, the possibility that the more general solutions will be obtained even if the travelling disturbance varies with the time and when the effect of a straight coast and the initial condition are considered. And thus we may say that we have advanced a step beyond the studies of these problems which many investigators have made hitherto.

In conclusion the writer wishes to express his sincere thanks to Prof. T. Nomitsu and Dr. T. Namekawa for their kind advice and encouragement during the study.

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